Fast Damping in Beam Envelope Oscillation Amplitudes of mismatched high intensity beams

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Abstract

Recently, a very fast damping of beam envelope oscillation amplitudes has been observed in simulations of high intensity beams transportation through periodic FODO cells, in mismatched conditions [1,2]. In those references, a guess on the possible mechanism that causes the damping effect has also been proposed. It seemed that the damping could be ascribed to a Landau damping mechanism.

In this presentation, further simulations, which seem confirm that the fast damping is due to the Landau damping effect, will be shown and discussed with more details.

1 INTRODUCTION

High-intensity charged-particle beams can develop extended low-density halos [3]. The existence of halos can have serious consequences in particle accelerators. If halo particles are lost in the accelerator, they may induce radioactivity. For the next generation of high intensity proton linac projects, it is necessary to obtain a more quantitative understanding of the physics of the halo. Multiparticle simulation studies of high intensity beams transported in linear focusing channels have provided some useful physical insights into the dynamics of high-current beams. For a non equilibrium beam injected with the correctly matched rms size, the initial distribution relaxes with accompanying emittance growth over a time of only about one quarter plasma period to a quasi equilibrium state with an approximately uniform central core in real space and with an edge that falls off over a distance approximately equal to the Debye length [4]. Furthermore, numerical simulations of rms-mismatched beams have shown that the beam envelope oscillations, due to mismatching, induce, on the beam, a halo, that could cause radiation activation on the surrounding [5]. Recently, multiparticle code simulations have shown that beam envelope oscillations, caused by mismatching with the periodic transport channel, can damp very fast [2]. In that reference, by making very rough assumptions on the envelope oscillation frequency, the hypothesis that this effect can be ascribed to a Landau mechanism of stabilization has been also presented.
In this work, more accurate assumptions on the envelope oscillations and a more detailed discussion on
the damping mechanism will be presented.

2 SIMULATIONS

In ref. [2], we made the very rough assumption that a high space charged beam, if mismatched with the periodic transport channel, oscillates at plasma frequency as reported in a ref. therein. This means that both odd and even mismatched beams should oscillate with the same frequency and then, since the odd mode of the case shown in [2] damps quickly also the even mode, for the same case, should damp in the same way. However, as shown in fig.1, we have verified that this is not true. This discrepancy will be accounted for in the following of the paper.
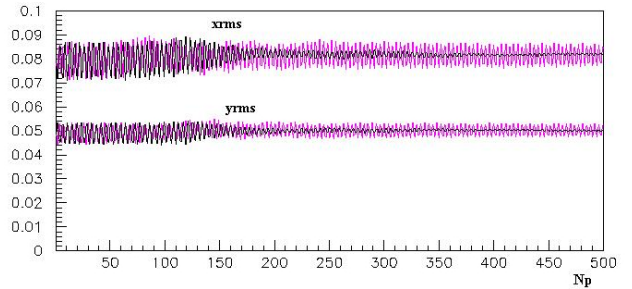

Fig.1: x and y rms-envelope oscillations for even (magenta-line) and odd mismatched beam (blak-line), with initial KV distribution, along the FODO channel.

In ref. [6], a more accurate theory for the envelope oscillations of mismatched beam show that it can be described by the equations :

$$z_1'' + k_1^2 z_1 = 0 \rightarrow k_1^2 = \left(\sigma_0^2 + 3\sigma^2\right)/S^2 \ (odd\ )$$

$$z_2'' + k_2^2 z_2 = 0 \rightarrow k_2^2 = \left(2\sigma_0^2 + 2\sigma^2\right)/S^2 \ (even\ )$$

where S, indicates the focusing periodic cell length, $z_1 = x(s) - y(s)$ and $z_2 = x(s) + y(s)$ are the odd and even mode of the rms envelopes $x(s)$ an $y(s)$ and s the longitudinal position along the periodic cell.
Notice that any arbitrary envelope oscillation mode can be expressed as a combination of these two fundamental modes (odd and even). The spatial frequencies k_1 and k_2 can be expressed as phase advances: $\Phi_1 = k_1 S$ and $\Phi_2 = k_2 S$. The phase advances obtained by this theory have been found in very good agreement with those given by the simulation results of the multiparticle code PARMT.
The main input file parameters, used in the PARMT simulations of ref. [2], used also for fig.1,are here recalled for sake of clarity. In that simulations, we used FODO cell

periods of length L=80 cm; transverse rms emittances, $\varepsilon_x=\varepsilon_y= 0.25 \times 10^{-6}$ m·r; single particle phase advance $\sigma_o=60.7°$ and tune depression $\sigma/\sigma_0=0.55$ corresponding to a beam current of 95 mA with an energy of 10 MeV and a 10% of mismatching on the beam size. As argued in that ref. , in these conditions, Landau damping can occur. In fact, there is a harmonic excitation given by the breathing mode oscillation of the beam envelope and a large set of oscillators, the beam particles oscillating with a large betatron frequency spectrum. Furthermore the coherent frequency must lay inside the betatron frequency spectrum. It can be noticed that the coherent oscillation energy is transferred to the incoherent oscillations of the beam particles. During the damping of the breathing oscillation there is a slight increase of the total emittance variation.

At the end, we can say that the energy involved in this process is dissipated from a coherent motion to the very high number of degrees of freedom of the system.

In the case of fig. 1, the odd rms envelope oscillation frequency is ω^o =76 MHz (from the theory of ref. [6] 78 MHz) while for the even mode ω^e =90 MHz. Then because the Landau damping could occur the frequency of the odd mode should lay inside the betatron frequency spectrum[2]. In fig. 2, it is shown a rough evaluation of the beam betatron frequency spectrum. It is an overlapping of the frequencies of some beam particles taken inside the beam at different positions along the beam radius. From the fig.2, it can be seen that half frequency of the odd oscillation mode is inside the betatron frequency spectrum while the other one, the even mode, is outside. This means that, following the arguments of ref [2], only for the odd mode the Landau damping can occur as shown in fig. 1. In the same fig. it can be noticed that the fast damping start after about 130 periodic cells. This is because, on the initial K-V particle distribution, before the Debye length tail is formed by the charge redistribution [4], all the beam particles undergo the same space charge tune shift and then oscillate with the same betatron frequency. In fact, in this case, the space charge force is linear. When the space charge forces are not enough to induce the Debye length tail on the beam spot the beam betatron spectrum is practically a kind of delta of Dirac. In the same ref. [2] it is shown that for $\sigma/\sigma_0=0.83$, no Landau damping occured, as expected, because the Landau damping conditions are no more matched. But, if we consider for the same $\sigma/\sigma_0=0.83$, an initial beam distribution of Gaussian type, the fast damping will occur again because the Landau damping conditions (large betaron frequency spectrum and coherent frequency located inside the spectrum) are again matched as shown in fig.3a).

Notice, in this last case, both the oscillation modes have
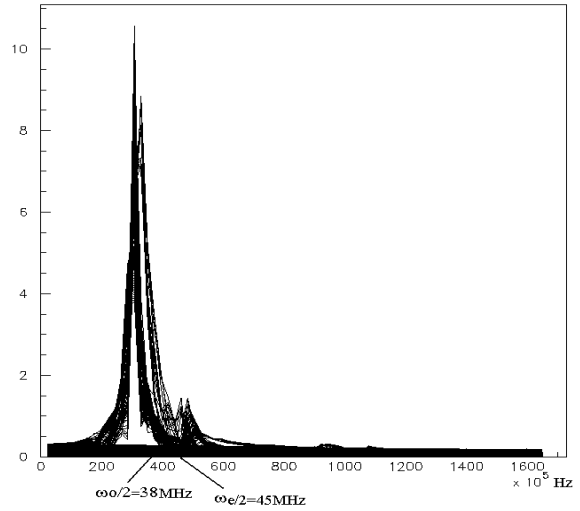

Fig. 2:Overlapping of FFT of some particle beam trajectories with initial x positions taken from 0.07 to 0.18 cm. The beam radius, after the Debye length tail is formed, is about 0.2.

frequencies laying inside the betatron spectrum and then both the modes quickly damp as shown in fig.3b).

To confirm that the fast damping observed in the simulations is due to a Landau mechanism of stabilization as argued in ref. [11], further simulations are considered for a different space charge parameter, that is a different σ/σ_0 , and also with different initial beam distributions.

In fig. 4 are shown the simulation results for $\sigma/\sigma_0=0.29$ (corresponding to a beam current, I=200 mA), with an initial beam particle distribution of KV type. In this case both the rms envelope oscillation modes are outside of the betatron spectrum as indicated in 4a) ($\omega_o/2=32$ MHz and $\omega_e/2=42$ MHz). Then , as expected from the above considerations on the damping mechanism, no damping can be observed on both the rms envelope oscillation modes shown in fig.4b).

In fig.5, instead, are shown the simulations for the same previous case, but with an initial beam distribution
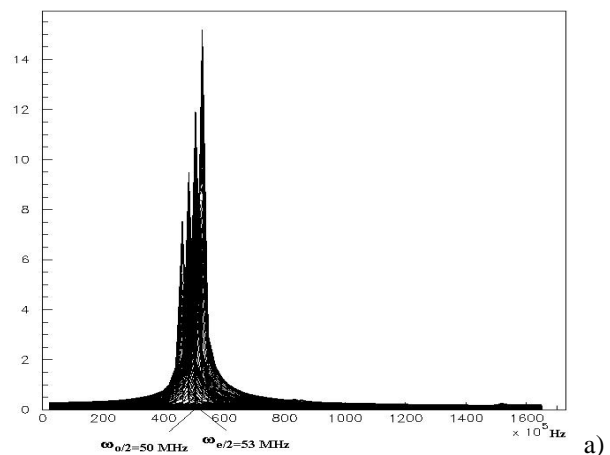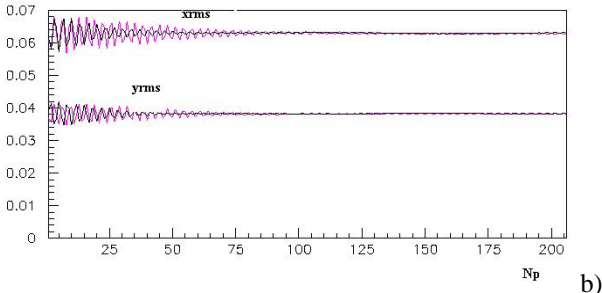

a)

Fig.3: a) The same of fig.2 for the case σ/σ_0=0.83 with initial Gaussian beam particle distribution. b) Beam rms envelope oscillations for the odd (black-line) and the even (magenta-line) mode for σ/σ_0=0.83.

of Gaussian type. This time, the betatron spectrum is larger and then the odd mode frequency is inside to it, as
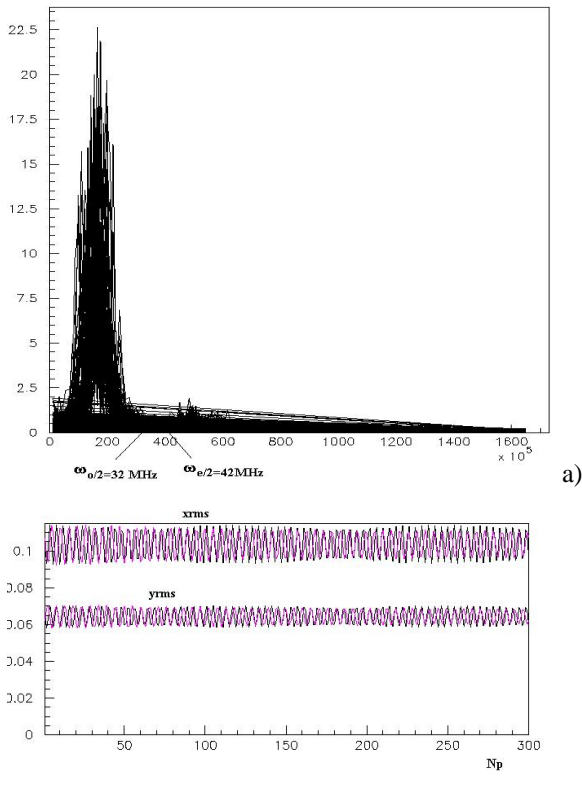

Fig.4: The same of fig.3, but for σ/σ_0=0.29, with initial K-V beam particle distribution.

it can be seen from the fig.5a). Then, as argued before, only the odd mode will undergo a fast damping as confirmed by the results shown in fig.5b).

CONCLUSION

Although, in this paper, a rough evaluation of the beam particle betatron spectrum has been considered, the determination of the main damping condition (that is oscillation mode frequency in or out the betatron spectrum) has been still assessed correctly. In fact, in our spectrum evaluation, we have considered particles at

different position from the beam center. Being, the space charge tune shift undergone by the particles, position dependent (if the space charge is not linear), in our calculation, we have considered, roughly, all the betatron frequency present in the beam.
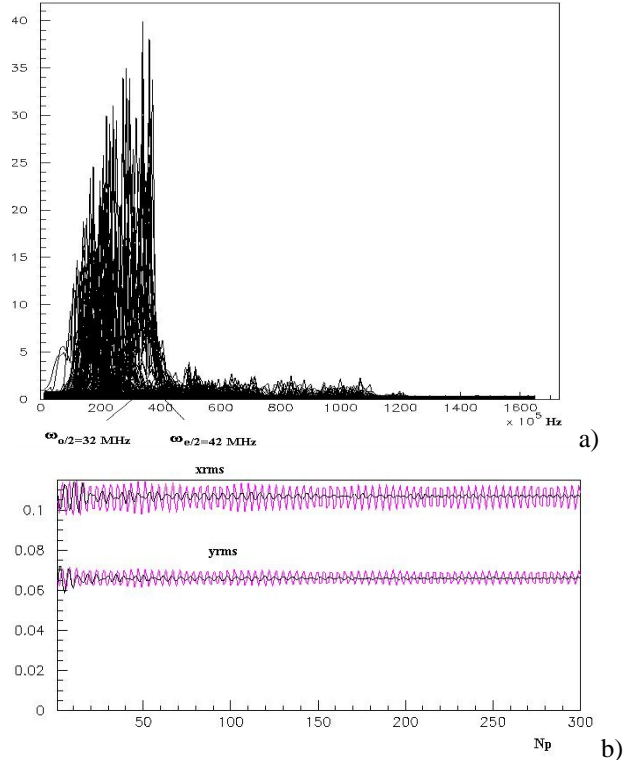

Fig.5: The same of fig.4, but with an initial Gaussian particle distribution.

Anyway a more precise evaluation of the betatron spectrum considering all the beam particles is underway.

In conclusion all the simulation results seem confirm that the damping effect shown in some cases are consequence of the Landau mechanism of stabilization through which a coherent oscillation (the envelope mode) transfer its energy to the many degree of freedom of an ensemble of oscillating systems (the beam particles oscillating at the betatron frequencies).

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